Pull-in instability often occurs when a film of a dielectric elastomer is subjected to an electric field. In this work, we concoct a set of simple, experimentally implementable, conditions that render the dielectric elastomer film impervious to pull-in instability for all practical loading conditions. We show that a uniaxially pre-stretched film has a significantly large actuation stretch in the direction perpendicular to the pre-stretch and find that the maximal specific energy of a dielectric elastomer generator can be increased from 6.3 J g⁻¹ to 8.3 J g⁻¹ by avoiding the pull-in instability.

Soft dielectrics are capable of achieving significantly large deformations and find application in humanlike robots, stretchable electronics, actuators, and energy harvesters among others. However, soft dielectrics under applied electric fields are also vulnerable to various types of electromechanical instabilities. Instabilities are often thought to be detrimental to material and device functionality and often avoided by design. Recent works have, however, focused on harnessing instabilities and find application in humanlike robots. Soft dielectrics are capable of achieving significantly large defor-
mations and find application in humanlike robots, actuation and energy harvesting.

Avoiding the pull-in instability of a dielectric elastomer film and the potential for increased actuation and energy harvesting

Shengyou Yang, Xuanhe Zhao and Pradeep Sharma

Cite this: Soft Matter, 2017, 13, 4552
DOI: 10.1039/c7sm00542c
Published on 16th March 2017. Downloaded by University of Houston on 05/07/2017 16:25:16.
We remark that the stretch \( \lambda_1 \), the dead load \( P_2 \), and the voltage \( \Phi \) in eqn (1) are prescribed parameters. In contrast to Yang et al.\textsuperscript{36} and Dorfmann and Ogden\textsuperscript{27} the dielectric film in this work admits only a class of homogeneous deformations. Thus, the general coordinate \( \lambda_1 \) has a zero variation \( \delta \lambda_1 = 0 \) for any homogeneous perturbation. When other two generalized coordinates \( \lambda_2 \) and \( \tilde{D} \) vary by small amounts \( \delta \lambda_2 \) and \( \delta \tilde{D} \), the free energy of the system varies by

\[
\frac{\delta G}{L_1 L_2 L_3} = \left( \frac{\partial W}{\partial \lambda_2} - s_2 \right) \delta \lambda_2 + \left( \frac{\partial W}{\partial \tilde{D}} - \tilde{E} \right) \delta \tilde{D} + \frac{1}{2} \frac{\partial^2 W}{\partial \lambda_2^2} \delta \lambda_2^2 + \frac{1}{2} \frac{\partial^2 W}{\partial \tilde{D}^2} \delta \tilde{D}^2 + \frac{\partial^2 W}{\partial \lambda_2 \partial \tilde{D}} \delta \lambda_2 \delta \tilde{D},
\]

where only the first and second variations are retained and all the high-order terms are omitted. At equilibrium, the first variation vanishes, and yields the following equilibrium equations:

\[
s_2 = \lambda_2 \left( 1 + \frac{\tilde{E}}{\mu} \lambda_2^{-2} \right), \quad \tilde{E} = \tilde{E},
\]

(3)

where the nominal stress \( s_2 = P_2/[L_1 L_3] \) and the nominal electric field \( \tilde{E} = \Phi/L_3 \) are prescribed parameters. In contrast, the nominal stress \( s_1 \) is defined as

\[
s_1 = \frac{\partial W}{\partial \lambda_1},
\]

(4)

which is no longer prescribed but depends on \( \lambda_2 \) and \( \tilde{D} \) as well as \( \lambda_1 \). Indeed, the partial derivative in eqn (4) is to be understood as the partial derivative of \( W \) with respect to \( \lambda_1 \) at the pre-stretch \( \lambda_1 \). It is exactly the coefficient of the zero \( \delta \lambda_1 \) in the first variation of the energy. Thus, it has been omitted in the variation form for simplicity. In equilibrium, from Cauchy’s stress theorem, \( s_1 \) in eqn (4) is equal to the stress generated by the force applied on the left and right surfaces, and the magnitude of the force is \( s_1 L_2 L_3 \).

From the principle of minimum energy, the stability of the equilibrium solution requires a positive-definite second variation in eqn (2) for arbitrary \( \delta \lambda_2 \) and \( \delta \tilde{D} \), that is, the Hessian matrix

\[
H = \begin{bmatrix}
\frac{\partial^2 W}{\partial \lambda_2^2} & \frac{\partial^2 W}{\partial \lambda_2 \partial \tilde{D}} \\
\frac{\partial^2 W}{\partial \lambda_2 \partial \tilde{D}} & \frac{\partial^2 W}{\partial \tilde{D}^2}
\end{bmatrix}
\]

must be positive definite for the equilibrium solution.

Consider an ideal dielectric elastomer with the free energy function:\textsuperscript{28,35}

\[
W(\lambda_1, \lambda_2, \tilde{D}) = \frac{\mu}{2} \left( \lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \lambda_2^{-2} - 3 \right) + \frac{\tilde{D}^2}{2 \tilde{E}} \lambda_1^{-2} \lambda_2^{-2},
\]

(6)

where \( \mu \) is the small-strain shear modulus and \( \epsilon \) is the permittivity. The first and second terms on the right-hand side of eqn (6) are the elastic and the dielectric energy. Then the equilibrium condition [eqn (3)] and the nominal stress \( s_1 \) in eqn (4) become

\[
s_2 = \mu \left( \lambda_2 - \left( 1 + \frac{\tilde{E}}{\mu} \lambda_2^{-2} \right) \lambda_2^{-2} \right), \quad \tilde{E} = \tilde{E},
\]

(7a)

and

\[
s_1 = \mu \left( \lambda_1 - \left( 1 + \frac{\tilde{E}^2}{\mu} \lambda_1^{-3} \lambda_2^{-2} \right) \lambda_2^{-2} \right).
\]

(7b)

Eqn (7) contains two algebraic equations with two variables \( \lambda_2 \) and \( \tilde{D} \) and three prescribed parameters \( \lambda_1, s_2, \) and \( \tilde{E} \). A quartic equation in terms of \( \lambda_2 \) can be obtained by eliminating \( \tilde{D} \), such that

\[
(\mu - \lambda_1^2 \lambda_2^2) \lambda_2^4 - s_2 \lambda_2^3 - \mu \lambda_2^2 = 0,
\]

(9)

which has only one positive real root of \( \lambda_2 \) if and only if

\[
0 \leq \frac{\tilde{E}}{\sqrt{\mu/\epsilon}} < \lambda_1^{-1},
\]

(10)

and the real root of \( \lambda_2 \) in eqn (9) is a real number between max \( \{s_2(\mu - \lambda_1^2 \lambda_2^2)\}^{-1}, \mu^{1/4} \lambda_1^{-1/2} (\mu - \lambda_1^2 \lambda_2^2)^{-1/4}\} \) and their sum. Hence the critical nominal electric field is given by

\[
\tilde{E}_* = \sqrt{\mu/\epsilon} \lambda_1^{-1}
\]

(11)

below which the equilibrium solution can exist. We note that under the condition [eqn (10)] with prescribed parameters \( \lambda_1 > 0, s_2 \geq 0, \) and \( \tilde{E} \geq 0, \) eqn (7) exhibits positive solutions for \( \lambda_2 \) and \( \tilde{D} \). Otherwise, eqn (7) has no solution for realistic physical situations that admit positive \( \lambda_2 \) and non-negative \( \tilde{D} \).
The electromechanical stability of the electrostatic system directly relates to the property of the Hessian matrix [eqn (5)] that, at equilibrium [eqn (7)], is given by

$$H = \begin{bmatrix}
\left[1 + 3 \left(1 + \frac{D}{k}\right) \lambda_1^{-1} \lambda_2^{-4} \right]
\frac{2D}{e} \lambda_1^{-2} \lambda_2^{-3} \\
\frac{2D}{e} \lambda_1^{-2} \lambda_2^{-3}
\end{bmatrix}.$$ (12)

The $1 \times 1$ principal minors of the Hessian matrix [eqn (12)] are the diagonal entries $H_{11} > 0$ and $H_{22} > 0$, and the only $2 \times 2$ principal minor is the determinant of the Hessian matrix [eqn (12)], which is always positive, namely

$$\det H = \lambda_1^{-4} \lambda_2^{-6} \left(4 \mu + \lambda_2^{-2} \lambda_1^{-3}\right) > 0$$ (13)

due to the fact that $\lambda_2 = \lambda_3/(L_1 L_3) \geq 0$. Since all principal minors are positive, the Hessian matrix $H$ [eqn (12)] is always positive-definite. A positive-definite Hessian matrix ensures the stability of the homogeneous deformation in equilibrium, because the dielectric film in equilibrium is in a state of minimum free-energy. Thus the pull-in instability never appears in this homogeneously deformed system loaded with a prescribed stretch, a dead load and an electric voltage under the condition [eqn (10)]. We emphasize that the stability is under the condition that the film is not buckled in the $X_d$ direction—we will return to this point later in the communication.

To further understand the behavior of the dielectric film in equilibrium, two special cases are discussed. Case I is a dielectric film at a prescribed stretch $\lambda_1 = 1$ under an electric field and several dead loads $s_d$, while case II is a dielectric film at a zero dead load $s_d = 0$ under an electric field and several prescribed stretches $\lambda_1$.

In Fig. 2, we plot the behavior of the dielectric film for case I, i.e. $\lambda_1 = 1$. Fig. 2(a) shows that the nominal electric field increases monotonically with the increase of the nominal electric displacement. The nominal electric field is bounded by eqn (11) and the critical nominal electric field for the nonexistence of equilibrium solutions at $\lambda_1 = 1$ is $E^* \sqrt{\varepsilon/e} = 1$. With a constant nominal electric field but an increase of the dead load $s_d$, the nominal electric displacement increases in Fig. 2(a), since a dead load leads to a reduced thickness but a larger area, thus resulting in a larger capacity (and charge). Fig. 2(b) shows the relation of the nominal electric displacement and the true electric field under several dead loads $s_d$.

In Fig. 2(c), the variation of the stretch $\lambda_2$ is plotted. Both the nominal electric field and the dead load $s_d$ can increase the stretch $\lambda_2$. At the critical value $E^* \sqrt{\varepsilon/e} = 1$, the stretch $\lambda_2$ increases to infinity and the thickness $\lambda_3 = \lambda_2^{-1}$ decreases to zero, which, of course, is impossible in reality since prior to such a blowup, electric breakdown will ensue due to the large true electric field or, alternatively, mechanical rupture will take place. The actuation stretch is defined as $\lambda_2/\lambda_{2p}$, where $\lambda_{2p}$ is the pre-stretch that exists due to the prescribed stretch $\lambda_1$ and the dead load $s_d$. At a prescribed stretch $\lambda_1 = 1$, the actuation stretch $\lambda_2/\lambda_{2p}$ under several dead loads $s_d$ is shown in Fig. 2(d).

The true stress $\sigma_1 = \lambda_1 s_d$ shown in Fig. 2(e) is directly related to electrical buckling and will be analyzed in the following. At a zero dead load in Fig. 2(e), the true stress is $\sigma_1/\mu = 1 - (1 - E^2/\mu)^{-1/2}$, and the nominal electric field induces a compressive state in the film, i.e. $\sigma_1 < 0$, and the magnitude $|\sigma_1|$ increases monotonically with the increase of the nominal electric field. On the other hand, at a zero electric field, a dead load $s_d$ expands the film ($\lambda_3 > 1$) and induces a tensile state i.e. $\sigma_1/\mu = 1 - \lambda_3^{-2} > 0$ in Fig. 2(e). Interestingly, there exists a competition between the electric field and the dead load due to their opposite effects on the stress $\sigma_1$. At a low electric field, the dead load makes the dielectric film extend within the plane. When the electric field increases, the stress $\sigma_1$ gradually decreases from a tensile stress ($\sigma_1 > 0$) to a compressive one ($\sigma_1 < 0$). Without considering electric breakdown (under a high true electric field) and rupture by stretch (at a high stretch $\lambda_2$), the continually increasing $|\sigma_1|$ of the compressive stress will finally make the dielectric film buckle.

Fig. 3 plots the behavior of the dielectric film for case II, i.e. $s_d = 0$. We note that eqn (11) gives the limit of the nominal electric field, for example, $E^* \sqrt{\varepsilon/e} = 1$ for a prescribed stretch $\lambda_1 = 1$, while it is 0.2 for $\lambda_2 = 5$ in Fig. 3(a), (c) and (e). The increase of the stretch $\lambda_1$ in Fig. 3(a) increases the film surface area, leading to higher capacity and gain of additional charge.
In other words, at a prescribed nominal electric field below $\tilde{E} \sqrt{\varepsilon/\mu}$, a larger stretch $\lambda_1$ corresponds to a higher nominal electric displacement. Fig. 3(b) shows the corresponding relation between the true electric field and the nominal electric displacement.

In Fig. 3(c), the increase of the stretch $\lambda_1$ in the length direction will make the film decrease its width (or the pre-stretch $\lambda_2$) at a zero electric field, pre-stretch but an electric field, on the other hand, will tend to make the film expand in-plane and exhibit a larger stretch ($\lambda_2$). At $\lambda_2 = 0$, the actuation stretch $\lambda_2/\lambda_{2p}$ under several prescribed stretches $\lambda_1$ is shown in Fig. 3(d).

The true stress from eqn (7) and (8) at $\lambda_2 = 0$ is obtained as $\sigma_1/\mu = \lambda_1 \lambda_2/\mu = \lambda_1^2 - \lambda_1^{-1}(1 - \lambda_1^{-2} \tilde{E}^2/\mu)^{-1/2}$. Without the electric field, the true stress $\sigma_1/\mu$ is $\lambda_1^2 - \lambda_1^{-4}$. When the electric field increases from zero, for example, at a prescribed stretch $\lambda_1 > 1$ in Fig. 3(e) and (f), the true stress $\sigma_1$ will decrease from a tensile stress ($\sigma_1 > 0$) to a compressive one ($\sigma_1 < 0$).

It is well-known that a thin film subjected to a lateral compression is easy to buckle. In our model, the deformation in the $X_1$ direction is controlled by two well-lubricated plates, and a compressive stress ($\sigma_1 < 0$) in the film can occur under some conditions (see Fig. 2(c), f, 3e and f for example). In the following, we will discuss electric buckling of a dielectric film subjected to electromechanical loads.

The special case, loss of tension, is of interest because it is a turning point for the compression-tension behavior of the dielectric film. The compressive stress can make the film buckle and should be avoided.\(^{1,2,6,37}\) When the nominal stress $\sigma_1$ in eqn (8) becomes zero, it is in the so-called state of loss of tension. From the equilibrium equations [eqn (7)], the nominal electric field at loss of tension is

$$\frac{\tilde{E}}{\sqrt{\varepsilon/\mu}} = \lambda_2^{-1}\sqrt{1 - \lambda_1^{-4}\lambda_2^{-2}}$$  \(\text{for } \lambda_1\lambda_2 \geq 1,\)  \(14\)

where $\lambda_2 = \frac{1}{2\mu \lambda_1} \left(\lambda_1 + \sqrt{\lambda_1^2 + 4\mu^2(\lambda_1^2 + \bar{f})}\right)$ and $\bar{f} = \pi^2(1/L_1/L_3)^2$.

At the state of loss of tension, if we, for example, continue to prescribe the stretch $\lambda_1$ and the dead load $\lambda_2$ but increase the nominal electric field, the stress $\sigma_1$ in eqn (8) will decrease from zero to negative and the film will be in a state of compression. Inspired by Euler's buckling of a long, slender, ideal column, we analyze here the electrical buckling of a dielectric film. Euler's formula for the buckling of a column with two fixed end supports is

$$F_c = \frac{4\pi^2 E^2 F^2}{L_3^2},$$  \(15\)

where $E^c$ is the critical compressive force, $E^e$ is the effective elastic modulus, $F$ is the area moment of inertia of the cross section, and $L_3$ is the length of the column. For a film of an incompressible neo-Hookean dielectric with shear modulus $\mu$ under the small-deformation assumption, the effective modulus is $E^c = 3\mu$ and $F^c = L_1^2 L_3^2/12$. Thus the critical compressive stress $f^c$ from eqn (15) is

$$f^c = \mu f^c = \frac{F_c}{L_1 L_3} = \frac{\mu \pi^2}{(L_1/L_3)^2}$$  \(16\)

It is assumed that the dielectric film begins to buckle when the magnitude of the compressive stress $\sigma_1 = \lambda_1\lambda_2 < 0$ in eqn (8) reaches $\bar{f}$ in eqn (16). Together with the equilibrium equations [eqn (7)], the critical nominal electric field for the electrical buckling can be expressed as

$$\frac{\tilde{E}}{\sqrt{\varepsilon/\mu}} = \lambda_2^{-1}\sqrt{1 + \bar{f}^{-1}\lambda_1^{-2} - \lambda_1^{-4}\lambda_2^{-2}},$$  \(17\)

where $\lambda_2 = \frac{1}{2\mu \lambda_1} \left(\lambda_1 + \sqrt{\lambda_1^2 + 4\mu^2(\lambda_1^2 + \bar{f})}\right)$ and $\bar{f} = \pi^2(1/L_1/L_3)^2$.

Compared with the length ($L_1$) and the width ($L_3$) of the film, the thickness ($L_3$) is often much smaller. Then the buckling stress $f^c$ in eqn (16) is very small, for example, $f^c < 0.1$ for a film with an aspect ratio $L_1/L_3 > 10$. Therefore, the critical nominal electric field in eqn (17) for buckling is very close to that in eqn (14) for the loss of tension.

Fig. 4 shows the critical electric fields for the loss of tension [eqn (14)] and electrical buckling [eqn (17)] in which the aspect ratio is chosen to be $L_1/L_3 = 10$. The difference between the two
The dielectric film with a critical stretch

the critical value in eqn (17), the electrical buckling occurs in

a high dead load.

the increase of the stretch

field will avoid the electrical buckling. On the other hand, if a

loss of tension occurs in the film, but the film will

buckle if the electric field increases.

For each curve in Fig. 4(a), there exists a peak that corre-

sponds to the maximum of the critical nominal electric field. At

that peak, any infinitesimal variation of the stretch \( \lambda_1 \) will make

the film buckle; however, the decrease of the nominal electric field

will avoid the electrical buckling. On the other hand, if a

point on the buckling curve is on the left-hand side of the peak,

the increase of the stretch \( \lambda_1 \) will avoid the electrical buckling,

while a point on the right-hand side of the peak, the electrical

buckling can be avoided by a decrease of the stretch \( \lambda_1 \). The

corresponding relation between the critical true electric field

and the stretch \( \lambda_1 \) is shown in Fig. 4(b).

A high actuation strain for an actuator driven by an electric field is desirable. Without an electric field, the pre-stretch \( \lambda_{2p} \)
due to the prescribed stretch \( \lambda_1 \) and the dead load \( s_2 \) can be
determined by eqn (7) with \( E = 0 \). The effects of \( s_2 \) and \( \lambda_1 \) on the

pre-stretch \( \lambda_{2p} \) are shown in Fig. 5(a). The pre-stretch is

\( \lambda_{2p} = \lambda_1^{-1/2} \) at \( s_2 = 0 \), while it is approximately equal to \( s_2/\mu \) at

d a high dead load.

Subjected to the electric field, the dielectric film thins down

and expands in the plane. When the electric field increases to

the critical value in eqn (17), the electrical buckling occurs in the
dielectric film with a critical stretch \( \lambda_{2c}^{\prime} \). The critical actuation

stretch is defined as \( \lambda_{2c}^{\prime}/\lambda_{2p} \). Fig. 5(b) plots the effects of the

stretch \( \lambda_1 \) and the dead load \( s_2 \) on the actuation stretch \( \lambda_{2c}^{\prime}/\lambda_{2p} \).

It shows that when the dielectric film is subjected to a pre-
scribed stretch \( \lambda_1 \), the actuation stretch in the direction normal
to the prescribed stretch is significantly larger, especially in the

case of a larger prescribed stretch \( (\lambda_1) \) and at a zero dead load

\( (s_2 = 0) \). Hence the actuation stretch can be dramatically

increased by a prescribed stretch. A similar observation has also

reported before in the analysis of the electromechanical

instability of a uniaxial pre-stressed dielectric film.\(^{28}\)

Inspired by the aforementioned discussion related to the

avoidance of pull-in instability, we now show the possibility of

increasing the capacity of the energy conversion of a dielectric

elastomer generator. It is known that the usual modes of failure

in a dielectric film include electrical breakdown (EB), electro-

mechanical instability (EMI or pull-in instability), loss of tension,

and rupture by stretch. The area of the cycle enclosed by these

four modes of failure is exactly the maximal energy that can be

converted in a dielectric film subjected to equal biaxial in-plane

forces.\(^{7}\) With the same dielectric film but mechanical boundary

conditions suggested in this work, the pull-in instability can be

avoided and then the four modes of failure reduce to three. This

reduction admits the possibility of enhanced energy conversion.

In the following, we will show that not only the maximal energy

of a dielectric elastomer generator but also the specific energy

enclosed by a rectangular in the voltage–charge plane and the

amplification of voltage (ratio of the input voltage to the output

voltage) can be increased significantly.

In a previous work,\(^{7}\) the dielectric film is subjected to equal

biaxial in-plane forces and voltage in the thickness direction,
such that the equal nominal stresses \( \sigma_1 = \sigma_2 \) and the equal

stretches \( \lambda_1 = \lambda_2 \) at equilibrium. It should be noted that there is

no difference between the forms of the equilibrium equations

in the work\(^{7}\) and in this paper, but the difference is the control

parameters. Unlike the equal biaxial in-plane forces,\(^{7}\) this paper

takes a prescribed stretch \( \lambda_1 \) and a dead load \( s_2 \) as control

parameters. Since the equilibrium equations have the same forms,

the equilibrium solutions of a film subjected to equal biaxial in-

plane forces – as discussed by Koh et al.\(^{7}\) – can be achieved by

choosing properly controlled parameters (\( \lambda_1, s_2 \)) in this work;

however, the pull-in instability will be avoided. This similarity

makes feasible the direct use of their model in this work for the

illustration of enhanced energy conversion by the proposed

avoidance of the pull-in instability.

To make this communication self-contained, we briefly

review the four modes of failure when subjected to equal

nominal stresses \( \sigma = \sigma_1 = \sigma_2 \) and equal stretches \( \lambda = \lambda_1 = \lambda_2 \) in

equilibrium [eqn (7)]. First, the curve under a zero electric field

\( (E = 0) \) in Fig. 6(a) is determined by eqn (7a) with \( D = 0 \), while it is

the origin in Fig. 6(b). Next, the electrical breakdown (EB)
curve is governed by eqn (7) with \( \tilde{E} = E_{\text{EB}} \lambda^{-2} \), where \( E_{\text{EB}} = 3 \times 10^{8} \text{ V m}^{-1} \) is

the critical true electric field when EB occurs.\(^{7,38}\) Other

material parameters used in the numerical calculations are

\( \mu = 10^{6} \text{ N m}^{-2} \) and \( \varepsilon = 3.54 \times 10^{-11} \text{ F m}^{-1} \) as well as the mass

density \( \rho = 1000 \text{ kg m}^{-3} \). Then, the curve of the loss of tension

is the horizontal axis in Fig. 6(a), while it is represented by

eqn (7) with \( s = 0 \) in Fig. 6(b). The curve of rupture by stretch in

Fig. 6(a) is the vertical line \( \lambda = \lambda_{\text{ru}} \) while in Fig. 6(b) it is
determined by eqn (7b) with \( \lambda = \lambda_{\text{br}} \) where \( \lambda_{\text{br}} \leq 6 \) when the film ruptures when subjected to equal biaxial stretch. Here we use \( \lambda_{\text{br}} = 5 \). Last, the curve of electromechanical instability (EMI) is based on eqn (6) and (7) in the work by Koh et al.\(^7\).

In Fig. 6, the shaded areas enclosed by various modes of failure (also the \( E = 0 \) curve) define the maximal energy of conversion, that is, the maximal specific energy. In the work by Koh et al.\(^7\) four modes of failure leads to a maximal specific energy of 6.3 J g\(^{-1}\). In contrast, the EMI (pull-in instability) is avoided in our proposed scheme and the remaining three modes of failure admit a maximal specific energy of 8.3 J g\(^{-1}\), increasing the capacity of the dielectric elastomer generator by nearly 33%.

The aforementioned maximal-energy cycle is idealized and may be difficult to realize in practice. The rectangular\(^2\) and triangular\(^3\) cycles are often used for energy conversion. We refer the reader to the Koh et al.\(^7\) for the detailed circuit design that pumps electric charge from a low-voltage battery to a high-voltage battery. A rectangle with vertices 1–2–3–4 is plotted in Fig. 6(b), where the energy enclosed by the rectangle is called the specific energy. The electric voltage corresponding to vertices (1,2) is the input voltage \( \Phi_{\text{in}} \) while the electric voltage corresponding to vertices (3,4) is the output voltage \( \Phi_{\text{out}} \). The specific energy and the amplification of voltage for various \( \Phi_{\text{in}} \) are plotted in Fig. 7(a) and (b). Evidently, the avoidance of pull-in instability can enhance the ability of energy conversion of a dielectric elastomer generator by increasing the specific energy and the voltage amplification.

In summary, in this work, we propose the avoidance of the pull-in instability of a dielectric film by introducing controlled-displacement boundary conditions, which ensure that the Hessian matrix is always positive definite in equilibrium regardless of the values assigned to the prescribed stretch, the dead load, and the electric field. The limit of the nominal electric field for the existence of equilibrium solutions is presented. We also show that the critical electric field for the loss of tension is slightly below that of electrical buckling and a uniaxial pre-stretched dielectric film can exhibit a significantly larger actuation strain in the direction perpendicular to the pre-stretch. Here we should emphasize that the film needs to be highly pre-stretched, uniaxially, to avoid loss of tension (or electrical buckling) when the electric field is high and the dead load is low. Finally, we find that the maximal specific energy that can be harvested may be increased from 6.3 J g\(^{-1}\) to 8.3 J g\(^{-1}\) by avoiding pull-in instability.

Acknowledgements
The authors gratefully acknowledge support from NSF CMMI Grant No. 1463205 and the M. D. Anderson Professorship.

References