Journal of Applied Mechanics

Book Review

Extending the Theory of Composites to Other Areas of Science, by Graeme Walter Milton, Milton-Patton Publishers, Salt Lake City, UT, 2016. ISBN: 978-1483569192

REVIEWED BY PRADEEP SHARMA, Department of Mechanical Engineering, Department of Physics, University of Houston, Houston, TX 77204.

We would be hard-pressed to find an area of research that permeates so deeply in nearly all the branches of physical sciences and mathematics as does the topic of *homogenization*.[1] The notion of studying an effective or apparent or overall behavior of a complex system that behaves differently microscopically occurs recurrently in all the branches of science and engineering. On the one hand, we can find a paper by Einstein, who derived the effective viscosity of a fluid dispersed with a dilute suspension of spherical particles, and on the other hand, the celebrated work by Clausius that discusses the apparent electrical permittivity of a dielectric composite. Those who understand the field know that these and many such relations from different fields can be derived in a unified manner. From understanding the effective mechanical properties of composites to designing metamaterials for cloaking, the field of homogenization has made tremendous strides over the past hundred plus years. While there has been some "cross-contamination," interestingly enough, much of the earlier development appears to have proceeded independently across different fields. This is not surprising since discipline-specific jargon tends to dampen a facile translation of ideas across fields. Mathematicians have developed their own nuanced version of the field and, to some extent, are perhaps also responsible for some of the interdisciplinary transfer.

Over the years, there have been several nice attempts to synthesize the field and present the concepts in a coherent fashion. A classical in this regard, in my opinion, is the tome by Graeme Milton, *Theory of Composites*. Although written by a mathematician (and I am not one), I always found it to be accessible to people like me who have an engineering background. This review is, however, about Milton's new book, *Extending the Theory of Composites to Other Areas of Science*.[2] The goal of this book is simple: *present a synthesis and distillation of the field of homogenization and then discuss a suite of scientific topics across which the key ideas can be utilized for enhanced insights*. The book succeeds wonderfully in this mission!

Chapters 1 and 2 of this book kick it off with a very elegant presentation of the abstract theory of composites. The abstract construction is worth the effort since it teases out the unification across the various scientific disciplines—be it elasticity or Schrodinger's formulation of quantum mechanics.[3] Like his previous book, I found this one to be quite accessible as well and was readily able to translate several of the chapters to subtopic I know relatively better (solid mechanics and elasticity).

Chapter 3 was quite new to me. Milton introduces here the idea that finding the so-called Dirichlet-to-Neumann map is analogous to finding the effective property of a composite material. This notion allows then the straightforward use of several tools and tricks of composites to tackle problems (in equal footing) related to acoustics, electromagnetism, and elastodynamics of heterogeneous materials. I would like to comment that while most of the chapters are authored by Milton, he is joined in a few by his collaborators: Maxence Cassier, Aaron Welters, Ornella Mattei, and Moti Milgrom.

With these three building blocks in hand (Chaps. 1–3), the book presents several interesting themes: a collection of chapters that present fresh approaches to standard composites theory, inverse problems (i.e., estimating material heterogeneities from boundary data), exploitation of composite theory insights for quantum mechanics, response of multifunctional materials (coupled fields), and several others. In the later chapters, the reader is likely to stumble upon several new results in diverse contexts but all united by the framework laid out in the beginning chapters. For example, in Chap. 6, which Milton co-authors with Ornella Mattei, I find a result that I have never seen before: bounds on the response of viscoelastic composites *at a specific time*. This result could potentially have profound implications for experimental characterization of microstructure of composites.

Like its precursor book (*Theory of Composites*), the new book is richly referenced and I particularly enjoyed the personable style in which the literature is discussed. In addition to an entirely novel pedagogical resource in its own class, this book also proves to be a good source of annotated history of the pertinent literature in this field.

I personally found inspiration for several new research directions even though it will take me a few more readings to fully appreciate all the deeper nuances[4] of the book.

[1]Depending on the discipline, this is also referred to as coarse-graining, upscaling, theory of composites, and so forth.

[2]The book is self-published via bookbaby™ and is available through the following link: https://store.bookbaby.com/book/Extending-the-Theory-of-Composites-to-Other-Areas-of-Science

[3]I am particularly appreciative of Milton's attempts to link quantum mechanics to the theory of composites. Where else is a mechanician likely to read about the link between the quantum simulation workhorse (density functional theory) and the topic that they are likely to be much more familiar with (homogenization)?

[4]Beyond the basic chapters, the reader may wish to pick and choose the themes to explore. For example, a few chapters are more focused on issues that are of interest to mathematicians (which I chose to skip in the first reading).

