Supplemental Information

Soft Matter Mechanics and the Mechanisms

Underpinning the Infrared Vision of Snakes

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First and Second Laws of Thermodynamics

Defining the dipole density per unit volume in the reference configuration by nominal polarization $\mathbf{P}$, the free energy density by $\Psi(\mathbf{F}, \mathbf{P}, T)$, and the local electric field by

$$\mathcal{E}[\mathbf{x}, \mathbf{P}] = \int_{\Omega_t} \frac{\varepsilon_0}{2} |\nabla_{\mathbf{x}} \xi|^2 = \int_{\Omega_R} \frac{\varepsilon_0}{2} J |\mathbf{F}^{-T} \nabla \xi|^2, \tag{1}$$

the total free energy of the body is:

$$A[\mathbf{x}, \mathbf{P}, T] = \int_{\Omega_R} \Psi(\mathbf{F}, \mathbf{P}, T) + \mathcal{E}[\mathbf{x}, \mathbf{P}]. \tag{2}$$

And the total internal energy of the body becomes

$$U[\mathbf{x}, \mathbf{P}, T] = \int_{\Omega_R} TS(\mathbf{F}, \mathbf{P}, T) + A[\mathbf{x}, \mathbf{P}, T], \tag{3}$$

where S is the local entropy density per unit volume in the reference configuration. We remark that here we have assumed that the body is homogeneous in space and time so Ψ, Φ, and S do not explicitly depend on $\mathbf{X}$. Considering Dirichlet boundary conditions

$$\begin{cases} \mathbf{x}(\mathbf{X}, t) = \mathbf{X} + \mathbf{u}_b(\mathbf{X}, t) & \text{on } \partial \Omega_R, \\ \xi(\mathbf{X}, t) = \xi_b(\mathbf{X}, t) & \text{on } \partial \Omega_R, \\ T(\mathbf{X}, t) = T_b(\mathbf{X}, t) & \text{on } \partial \Omega_R, \end{cases} \tag{4}$$

where $\mathbf{u}_b$, ξ_b, and T_b are the prescribed boundary displacement, prescribed electric potential, and prescribed temperature accordingly. The rate of work done to the system with no body force is

$$P_{\text{ext}} = \int_{\partial \Omega_R} (\dot{\mathbf{u}}_b \cdot \tilde{\mathbf{t}}^e - \xi_b \mathbf{D} \cdot \mathbf{n}), \tag{5}$$

where $\tilde{\mathbf{t}}^e$ is the traction applied to the system and $\mathbf{D} = -\varepsilon_0 J \mathbf{C}^{-1} \nabla \xi + \mathbf{F}^{-1} \mathbf{P}$ is the nominal electric displacement. The rate of total heat transferred into the body is

$$\dot{Q} = \int_{\Omega_R} r_e - \int_{\partial \Omega_R} \mathbf{q} \cdot \mathbf{n}, \tag{6}$$

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where $\mathbf{q}$ is the thermal flux and r_e is the rate of heat supply. Assuming that the considered system is stationary and neglect kinetic energy, first and second laws of thermodynamic imply that:

$$\frac{d}{dt}U = P_{\text{ext}} + \dot{Q},$$
$$\frac{d}{dt}\int_P S \geq -\int_{\partial P} \frac{\mathbf{q} \cdot \mathbf{n}}{T} + \int_P \frac{r_e}{T}, \tag{7}$$

where $P \subset \Omega_R$ is any material volume element in the reference configuration. With some tedious calculations addressed in Darbaniyan $et\ al.$,[1] we have

$$\frac{d}{dt}\mathcal{E} = \int_{\Omega_R} (\nabla \dot{\mathbf{x}} : \boldsymbol{\Sigma}_{\text{MW}} + \nabla \xi \cdot \mathbf{F}^{-1}\dot{\mathbf{P}}) - \int_{\partial\Omega_R} \xi_b \dot{\mathbf{D}} \cdot \mathbf{n}. \tag{8}$$

Where $\boldsymbol{\Sigma}_{\text{MW}} = (\mathbf{F}^{-T}\nabla\xi) \otimes (\varepsilon_0 J\mathbf{C}^{-1}\nabla\xi - \mathbf{F}^{-1}\mathbf{P}) - \frac{\varepsilon_0}{2}J|\mathbf{F}^{-T}\nabla\xi|^2\mathbf{F}^{-T}$ is the so called the Maxwell stress. Substituting in first and second laws of thermodynamic and rearranging, the standard Coleman-Noll approach implies that

$$\begin{cases} \dfrac{\partial\Psi}{\partial\mathbf{P}} = -\mathbf{F}^{-T}\nabla\xi & \text{in } \Omega_R, \\[2mm] S(\mathbf{F},\mathbf{P},T) = -\dfrac{\partial}{\partial T}\Psi(\mathbf{F},\mathbf{P},T) & \text{in } \Omega_R, \\[2mm] -T\dfrac{d}{dt}\dfrac{\partial\Psi}{\partial T} - r_e + \nabla \cdot \mathbf{q} = 0 & \text{in } \Omega_R, \\[2mm] \nabla \cdot \boldsymbol{\sigma} = 0 & \text{in } \Omega_R, \\[2mm] \boldsymbol{\sigma} \cdot \mathbf{n} = \tilde{\mathbf{t}}^e & \text{on } \partial\Omega_R. \end{cases} \tag{9}$$

Where $\boldsymbol{\sigma} = \dfrac{\partial\Psi}{\partial\mathbf{F}} + \boldsymbol{\Sigma}_{\text{MW}}$ is the total stress applied to the system and with the assumption of the linear Fourier's law: $\mathbf{q} = -k\nabla T$, with $k \geq 0$.

Constitutive Relations

In this section we specify free energy function $\Psi(\mathbf{F},\mathbf{P},T)$ that conserves the principle of frame indifference and material symmetry. If $SO(3)$ be the group including all rigid rotations and $\mathcal{G} \subset SO(3)$ be the subgroup associated with material symmetry, for any $\mathbf{R} \in SO(3)$ and $\mathbf{Q} \in \mathcal{G}$, we should have

$$\Psi(\mathbf{RF},\mathbf{RP},T) = \Psi(\mathbf{F},\mathbf{P},T), \ \ \Psi(\mathbf{FQ},\mathbf{P},T) = \Psi(\mathbf{F},\mathbf{P},T). \tag{10}$$

Choosing the reference configuration as the equilibrium one with $(\mathbf{P},T) = (\mathbf{0},T_0)$ in the absence of external loadings, we have $\Psi(\mathbf{F},\mathbf{P},T_0) \geq \Psi(\mathbf{I},\mathbf{0},T_0)$, with $\mathbf{I}$ being the identity matrix. We assume that the material is isotropic with permittivity ε independent of temperature and deformation and incompressible at constant temperature. With the latter

assumption, thermoelastic incompressibility, the volume change caused by thermal expansion has to be taken into account.[2] To that end having α as the linear thermal expansion coefficient and $\Delta T = T - T_0$, we assume that:

$$J = 1 + 3\alpha\Delta T. \tag{11}$$

Considering Ψ_0 as a constant identified as the free energy density at $(\mathbf{F}, \mathbf{P}, T) = (\mathbf{I}, 0, T_0)$, one possible form for free energy density can be

$$\Psi(\mathbf{F}, \mathbf{P}, T) = \Psi_0 - CT \log\frac{T}{T_0} + \frac{\mu}{2}(\mathbf{I}_1 - 3) - \Pi|J - (1 + 3\alpha\Delta T)| + \frac{|\mathbf{P}|^2}{2J(\varepsilon - \varepsilon_0)}, \tag{12}$$

where Π is the Lagrange multiplier and ε_0 is the vacuum electric permittivity. From Eq.(9), we get:

$$\begin{cases} \nabla \cdot [\mu\mathbf{F} - \Pi J\mathbf{F}^{-T} - \dfrac{\varepsilon}{2}J|\mathbf{F}^{-T}\nabla\xi|^2\mathbf{F}^{-T} + \varepsilon J(\mathbf{F}^{-T}\nabla\xi) \otimes (\mathbf{C}^{-1}\nabla\xi)] = 0 & \text{in } \Omega_R, \\ [\mu\mathbf{F} - \Pi J\mathbf{F}^{-T} - \dfrac{\varepsilon}{2}J|\mathbf{F}^{-T}\nabla\xi|^2\mathbf{F}^{-T} + \varepsilon J(\mathbf{F}^{-T}\nabla\xi) \otimes (\mathbf{C}^{-1}\nabla\xi)] \cdot \mathbf{n} = \tilde{\mathbf{t}}^e & \text{on } \partial\Omega_R, \\ C\dot{T} + 3\alpha T\dot{\Pi} = r_e + \nabla \cdot (k\nabla T) & \text{in } \Omega_R. \end{cases} \tag{13}$$

References

[1] F. Darbaniyan, K. Dayal, L. Liu, P. Sharma, Designing soft pyroelectric and electrocaloric materials using electrets, Soft matter.

[2] K. Volokh, Mechanics of soft materials, Springer, 2016.